Non Uniform Flow in Open channels

Learning Objectives

At the end of this topic, you will be able to:

→ Non-uniform flow through open channels.

→ Dynamic Equation for steady Gradually Varied flow.

→ Classification of channel bottom slopes.

→ Hydraulic Curve classifications

→ Hydraulic jump

→ Applications of hydraulic jump

→ Depth of hydraulic jump

→ Energy dissipation due to Hydraulic jump.

Learning Outcomes

By the end of this topic you will be able to,

→ Describe the various types of non-uniform flows.

→ Derive the dynamic equation for the steady gradually varied flow.

→ Describe the classification of channel bottom slopes.

→ Illustrate the classifications of Hydraulic curve

→ Elaborate the functions of Hydraulic jump.

→ find the various applications of hydraulic jump

→ find out the depth of hydraulic jump

→ find the reason for energy dissipation due to hydraulic jump.

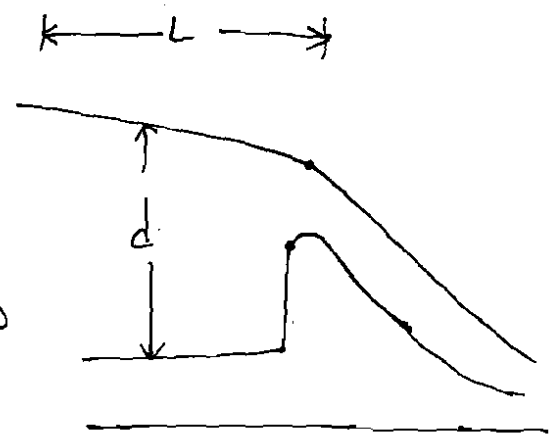
Non-uniform flow through open channels

→ Non-uniform flow is also known as the flow of varying depth & the varied flow. The varied flow may be defined as:

Gradually varied flow (GVF):-

- A steady non-uniform flow in a prismatic channel with gradual changes in its water surface elevation is termed as gradually-varied flow (GVF).

Steady, Gradually varying flow ($L > d$)



water surface profile behind a dam

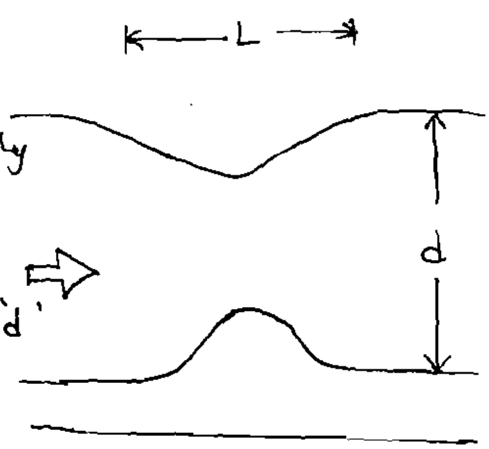
- The back water produced by a dam or weir across a river and the drawdown produced at a sudden drop in a channel are few typical examples of gradually-varied flow (GVF).
- In a GVF, the velocity varies along the channel and consequently the bed slope, water surface slope, and energy slope will all differ from each other.

Rapidly-varied flow (RVF):-

- If water depth or velocity change Abruptly over a short distance and the pressure distribution is not hydrostatic, The water surface profile is characterized as rapidly varying flow (RVF).
- The occurance of RVF is usualy a local phenomenon.

Steady, rapidly varying flow

'L comparable to d'



Flow over a short hump

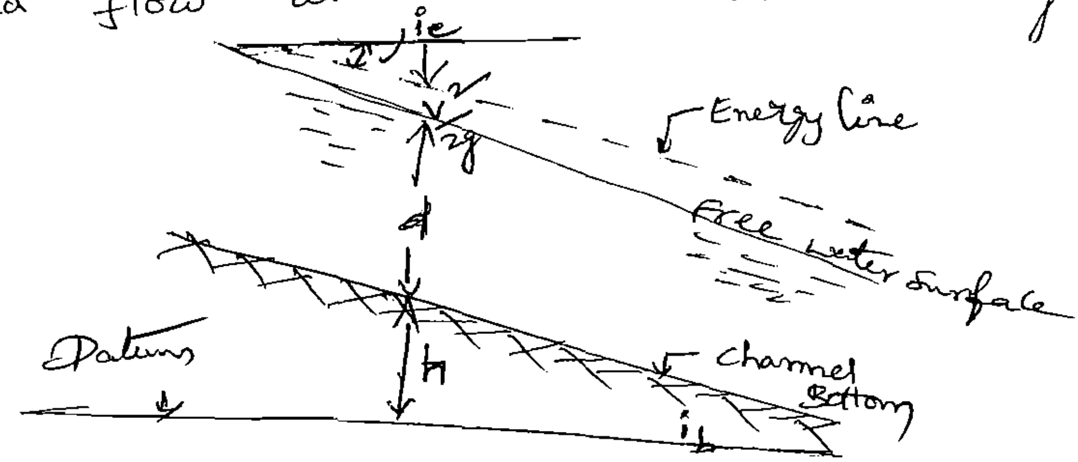
- RVF can often be observed near the inlet and outlet of culverts, and wherever hydraulic jumps occurs.

Dynamic Equation for Steady Gradually Varied Flow.

→ The dynamic equation for gradually varied flow can be derived from the basic energy equation with the following assumptions.

i) In a channel, the slope of the bed is small.
ii) The flow is steady and hence discharge Q, is constant
iii) The effects of acceleration will be highly negligible and so that the hydrostatic pressure distribution must prevails over through the channel cross-section
iv) The energy correction factor α is unity.

→ Let's consider a rectangular channel having gradually varied flow which as shown in the given figure



Let z = height of bottom of channel above datum
d = depth of flow
V = mean velocity of flow
i_b = Slope of the channel bed
i_e = Slope of the energy line
b = Width of channel and
Q = Discharge through the channel,

→ The energy equation ~~at~~ any Section is defined by Bernoulli's equation,

$$E = \cancel{Z} + d + \frac{V^2}{2g} \longrightarrow (1)$$

→ Differentiating each term of the above equation with respect to x, where x is measured along the channel bottom, the following differential equation can be obtained.

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dd}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \text{ ——— } (2)$$

Now $\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dx}\left(\frac{Q^2}{A^2 \times 2g}\right)$ $\left(\because V = \frac{Q}{A} = \frac{Q}{b \times d}\right)$

$$= \frac{d}{dx}\left(\frac{Q^2}{b^2 d^2 \times 2g}\right) = \frac{Q^2}{b^2 \times 2g}\frac{d}{dx}\left(\frac{1}{d^2}\right)$$

$(\because Q, b, g$ are constant$)$

$$= \frac{Q^2}{b^2 \times 2g}\frac{d}{dd}\left[\frac{1}{d^2}\right]\frac{dd}{dx}$$

$$= \frac{Q^2}{b^2 \times 2g}\left[\frac{-2}{d^3}\right]\frac{dd}{dx} = -\frac{2Q^2}{b^2 2g d^3} \cdot \frac{dd}{dx}$$

$$= \frac{-Q^2}{b^2 d^2 \times g d} \cdot \frac{dd}{dx} = -\frac{V^2}{gd} \cdot \frac{dd}{dx}$$

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dd}{dx} - \frac{V^2}{gd} \cdot \frac{dd}{dx}$$

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dd}{dx}\left[1 - \frac{V^2}{gd}\right] \longrightarrow (3)$$

Now $\frac{dE}{dx}$ = slope of the energy line $= -i_e$

and $\frac{dZ}{dx}$ = Slope of the bed of the channel $= -i_b$.

Negative sign with i_e and i_b is taken as with the increase of x, the value of E and Z decre[ase]. Substituting the above values in equation ③, we g[et]

$$\frac{dd}{dx} = \frac{i_b - i_e}{1 - \frac{V^2}{gd}} = \frac{(i_b - i_e)}{\left[1 - (F_e)^2\right]} \quad \left(\because \frac{V}{\sqrt{gd}} = F_e\right)$$

As 'h' is the depth of flow and x is the distan[ce] measured along the bottom of the channel hence represents the variation of the water depth along the bottom of the channel. So this is also called the slope of the free water surface.

Thus:

i, when $\frac{dd}{dx} = 0$, d is depth of the water above the bottom of Channel is constant. It means that free surface of water is parallel to the bed of the channel.

ii, when $\frac{dd}{dx} > 0$ & $\frac{dd}{dx}$ is +ve, it means the depth of water increases in the direction of flow.

iii, when $\frac{dd}{dx} < 0$ & $\frac{dd}{dx}$ is -ve, it means the depth of water decreases in the direction of flow.

Classification of channel bottom slopes

Steep slope :

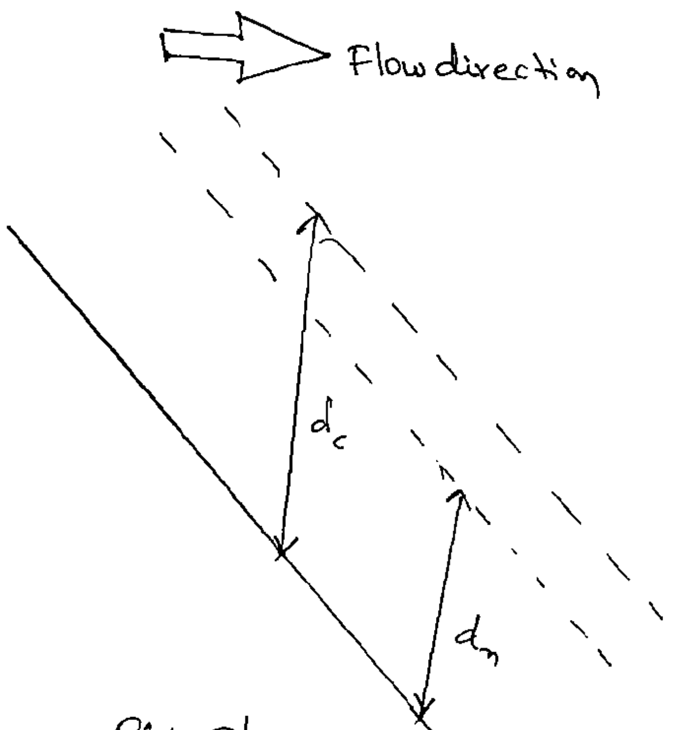


Fig: Slope increased substantially Steep slope

→ The channel bottom slope is designated as steep when the bottom slope S_b is greater than the critical slope.

i.e, $S_b > S_c$

→ Again the application of manning's (&) chezy's formula will indicate that when the bottom slope is steep, the normal depth of flow is less than the critical depth, i.e, $d_n < d_c$

Horizontal slope :

→ When the channel bottom slope is equal to zero i.e, $S_b = 0$, the bottom slope is designated as horizontal.

→ Obviously for a channel with horizontal bottom the normal depth of flow, $d_n = \infty$ (infinity)

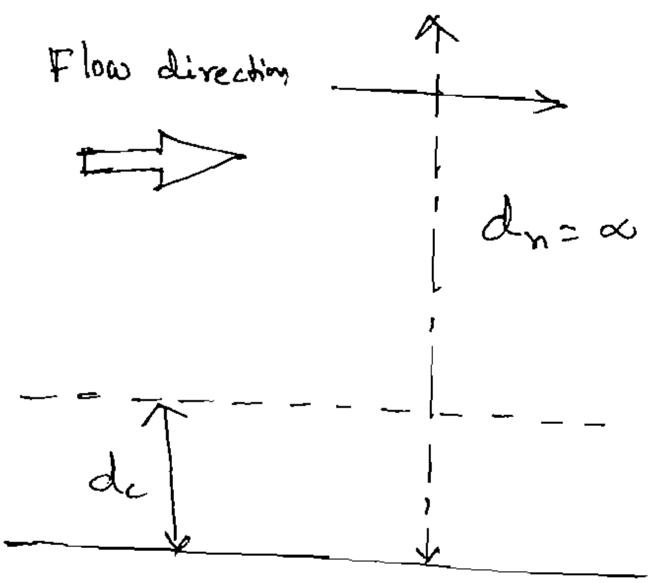


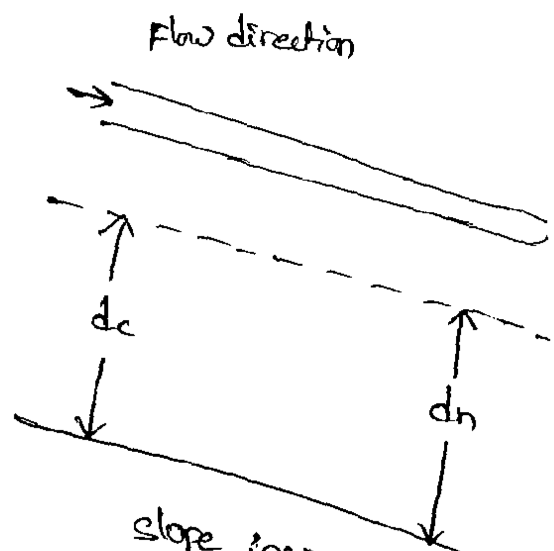
Fig: Normal depth (d_n) and critical depth (d_c) on horizontal sloped bed

Classification of channel bottom slopes:-

The different types of channel bottom slopes are classified into under various conditions as mentioned below.

Critical slope:-

The channel bottom slope is designated as critical when the bottom slope S_b is equal to the critical slope S_c. i.e $S_b = S_c$. Thus in case the normal depth of flow will be equal to the critical depth, i.e $d_n = d_c$



slope increased Further critical slope.

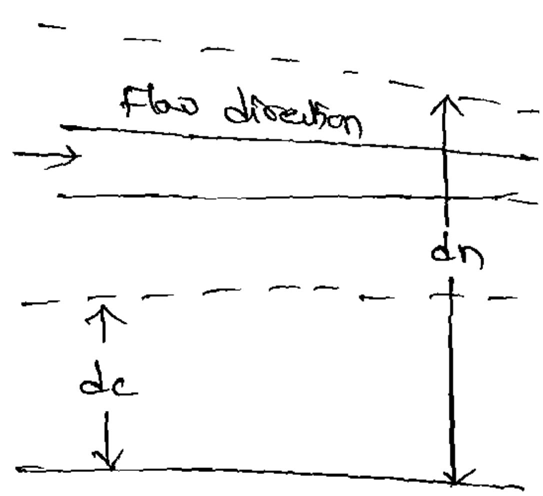
Mild slope:-

- The channel bottom slope is designated as mild when the bottom slope S_b is less than to the critical slope S_c.

  i.e $S_b < S_c$

- The application of Manning's or Chezy's formula will then indicate that when the bottom slope is mild, the normal depth of flow is greater than the critical depth

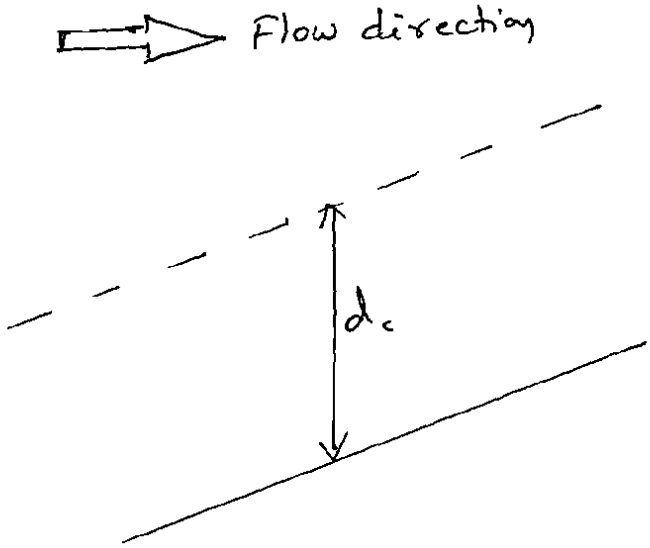
  i.e $d_n > d_c$



slope increased slight mild slope.

Adverse Slope.

→ When the channel bottom slope instead of falling rises in the direction of flow it is designated as an adverse slope. Thus in a channel with adverse bottom slope, S_b is less than zero i.e., $(S_b < 0)$ or it is negative.



Adverse slope. No defon d_n.

→ Obviously for an adverse-slopped channel the normal depth of flow d_n is imaginary or it is non-existent.

Hydraulic Curve classifications

→ Hydraulic Curve classifications are used to describe the shape of the water surface profile at a specific flow.

→ The curves are based on the Hydraulic slope (A, H, C, M, or S) and the relative position of the actual flow depth to normal and critical depth as designated by the numbers 1, 2 and 3.

1. Type 1 Curve: Depth is greater than d_c and d_n, flow is subcritical
2. Type 2 Curve: Depth is between d_c and d_n, flow can be either subcritical & supercritic
3. Type 3 Curve: Depth is less than both d_c and d_n

Hydraulic Jump

→ A hydraulic jump is a phenomenon in the science of hydraulics which is frequently observed in open channel flow such as rivers and spillways.

→ When liquid at high velocity discharges into a zone of lower velocity, a rather abrupt rise occurs in the liquid surface.

→ The rapidly flowing liquid is abruptly slowed and increases in height, converting some of the flow's initial kinetic energy into an increase in potential energy, with some energy irreversibly lost through turbulence to heat.

→ In order to study the conditions of flow before and after the hydraulic jump the application of the energy equation does not provide an adequate means of analysis, because hydraulic jump is associated with an appreciable loss of energy which is initially unknown.

Applications of Hydraulic Jump

1. To dissipate energy in water flowing over hydraulic structures as dams, weirs and others to prevent scouring downstream structures.
2. To raise water level on the downstream side for irrigation or other water distribution purposes.
3. To indicate special flow conditions such as existence of supercritical flow or the presence of a control section so that a gaging station may be located.
4. To mix chemicals used for water purification.
5. To aerate water for city water supplies.
6. To remove air pockets from water supply lines & prevent air locking.

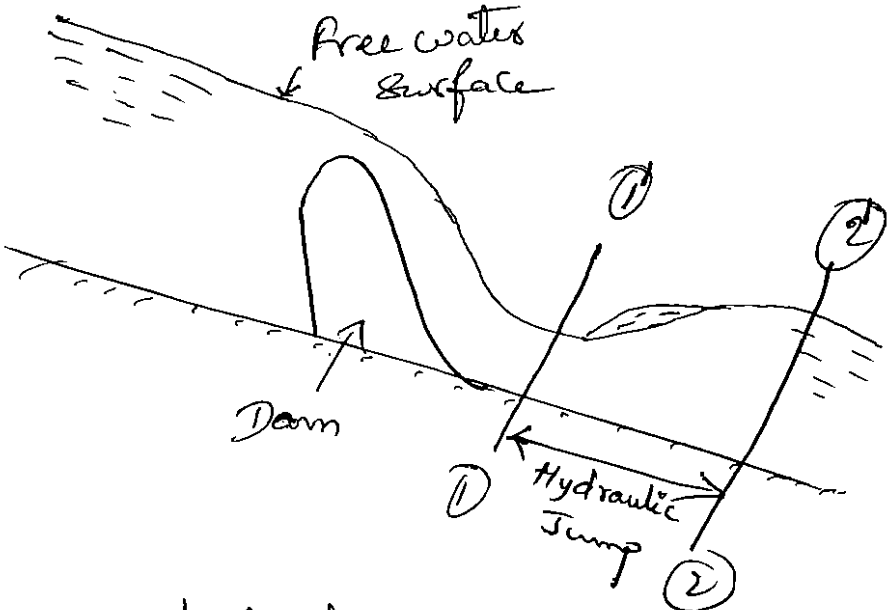
Depth of Hydraulic Jump:

→ As such in the analysis of hydraulic jump the momentum equation is used by considering the portion of the hydraulic jump as the control volume. The following assumptions are, however, made in this analysis

i) It is assumed that before and after jump formation the flow is uniform and the pressure distribution is hydrostatic.

ii) The length of the jump is small so that the losses due to friction on the channel floor are small and hence neglected.

(iii) The channel floor is horizontal & the slope is so gentle that the weight component of the water mass comprising the jump is negligibly small.



→ consider a hydraulic jump formed with a channel of horizontal bed as shown in the given figure

→ Let us consider the two sections 1-1' and 2-2' before and after hydraulic jump.

d_1 = Depth of flow at section 1-1'

d_2 = Depth of flow at section 2-2'

V_1 = Velocity of flow at section 1-1'

V_2 = Velocity of flow at section 2-2'.

D_1 = Depth of Centroid area at section 1-1' below free surface

D_2 = Depth of Centroid area at section 2-2' below free surface.

A_1 = Area of cross-section at section 1-1'

A_2 = Area of Cross-Section at Section 2-2'.

To consider unit width of the channel

The forces acting on the mass of water between sections 1-1' and 2-2' are:

(1) On Section 1-1', pressure force to be consider as P_1

(2) On Section 2-2', Pressure force to be consider as P_2

(3) Frictional force on the floor of the channel, which assumed to be neglegible.

Let,

$$q = \text{Discharge per unit width}$$

$$= v_1 d_1 = v_2 d_2 \longrightarrow (1)$$

Now. the pressure force, P_1 on Section 1-1'

$$\rho g A_1 D_1 = \rho g \times d_1 \times 1 \times \frac{d_1}{2}$$

$$= \frac{\rho g\, d_1^2}{2}$$

$$\left[\because A_1 = d_1 \times 1,\quad D_1 = \frac{d_1}{2}\right]$$

Similarly pressure force on section 2-2'.

$$P_2 = \rho g A_2 D_2$$

$$= \rho g \times d_2 \times 1 \times \frac{d_2}{2}$$

$$= \frac{\rho g\, d_2^2}{2}$$

$\therefore$

Net force acting on the mass of water between sections 1-1' and 2-2' $= P_2 - P_1$

$\because P_2$ is greater than P_1 and d_2 is greater than d_1

$$= \frac{\rho g d_2^2}{2} - \frac{\rho g d_1^2}{2} = \frac{\rho g}{2}\left[d_2^2 - d_1^2\right] \longrightarrow (2)$$

On based the momentum principle, the net force acting on a mass of fluid must be equal to the rate of change of momentum in the same section.

$\therefore$ Rate of change of momentum in the direction of force. = mass of water per sec $\times$ change of velocity in direction of force

Now mass of water per second $= \rho \times$ discharge per unit width $\times$ width

$$= \rho \times q \times 1$$

$$= \rho q \ \text{kg/s}$$

Change of velocity in the direction of force $= (V_1 - V_2)$

Assume that the net force is acting from right to left, then the change of velocity will be appear from the right to left and hence ~~is~~ equal to $(V_1 - V_2)$.

∴ Rate of change of momentum in the direction of force $= \rho q (V_1 - V_2)$ ⟶ (3)

Hence according to momentum principle, the expression given by equation (2) is equal to expression given by equation (3).

$$\therefore \frac{\rho g}{2}(d_2^2 - d_1^2) = \rho q (V_1 - V_2)$$

But from equation (1), $V_1 = \frac{q}{d_1}$ and $V_2 = \frac{q}{d_2}$

$$\therefore \frac{\rho g}{2}(d_2^2 - d_1^2) = \rho q \left[\frac{q}{d_1} - \frac{q}{d_2}\right]$$

$$\frac{\cancel{\rho} g}{2}(d_2^2 - d_1^2) = \cancel{\rho} q^2 \left[\frac{d_2 - d_1}{d_1 d_2}\right]$$

$$\frac{g}{2}(d_2 + d_1)\cancel{(d_2 - d_1)} = q^2 \left(\frac{\cancel{d_2 - d_1}}{d_1 d_2}\right)$$

$$\frac{g}{2}(d_2 + d_1) = \frac{q^2}{d_1 d_2}$$

$$(d_2 + d_1) = \frac{2q^2}{g(d_1 d_2)} \longrightarrow (4)$$

multiplying both the sides by d_2, we get

$$d_2^2 + d_1 d_2 = \frac{2q^2}{g d_1} \quad \text{(or)} \quad d_2^2 + d_1 d_2 - \frac{2q^2}{g d_1} = 0 \longrightarrow (5)$$

Equation ⑤ is a quadratic equation in d_2 and hence its solution is

$$d_2 = \frac{-d_1 \pm \sqrt{d_1^2 - 4\times 1\left(-\frac{2q^2}{gd_1}\right)}}{2}$$

$$= \frac{-d_1 \pm \sqrt{d_1^2 + \frac{8q^2}{gd_1}}}{2}$$

$$= -\frac{d_1}{2} \pm \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

The two roots of the equation are

$$d \text{ ⑧ } d_2 = -\frac{d_1}{2} - \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} \quad \text{and} \quad -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

First root is not possible as it gives −ve depth.

Hence $d = -\frac{d_1}{2} - \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$ $(\because q = v_1 d_1)$

d ⑧ d_2 $\quad d = -\frac{d_1}{2} - \sqrt{\frac{d_1^2}{4} + \frac{2(v_1 d_1)^2}{gd_1}}$

&

Depth of hydraulic Jump $\underline{d} = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2\times v_1^2 d_1}{g}}$

$\therefore$ Height ~~Depth~~ of hydraulic Jump $= (d_2 - d_1)$

Problem

1. The depth of flow of water, at a certain section of a rectangular channel of 6m wide, is 1m. This discharge through the channel is $12\ m^3/s$. If a hydraulic jump takes place on the downstream side. Find the depth of flow after the jump.

Given Data:

Width of channel, $b = 6m$

Depth of flow before jump, $d_1 = 1m$

Discharge, $Q = 12\ m^3/s$

Discharge per unit width, $q = \frac{Q}{b} = \frac{12}{6} = 2\ m^2/s$

To find:

Depth of flow after the jump

Formula used:

$$d_2 \text{ (or) } d = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{g d_1}}$$

Solution:

Let the depth of flow after jump $= d_2$ (or) d

Depth of flow after jump is given by

$$d_2 \text{ (or) } d = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{g d_1}} = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2 \times 4}{9.81}} = 0.5322m$$

Result: Depth of flow after the jump, $d_2 = 0.5322\, m$.

⇒ Energy Dissipation due to Hydraulic Jump:

→ When the hydraulic jump takes place, a loss of energy will be appeared due to the eddies formation and turbulence actions. So this energy loss is equal to the difference of specific energies at sections 1-1' and 2-2'.

$$h_L = E_1 - E_2$$

$$= \left(d_1 + \frac{V_1^2}{2g}\right) - \left(d_2 + \frac{V_2^2}{2g}\right) \qquad \left[\because E_1 = d_1 + \frac{V_1^2}{2g} \quad \because E_2 = d_2 + \frac{V_2^2}{2g}\right]$$

$$= \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - (d_2 - d_1)$$

$$= \left[\frac{q^2}{2g d_1^2} - \frac{q^2}{2g d_2^2}\right] - (d_2 - d_1) \qquad \left(\because V_1 = \frac{q}{d_1} \text{ \& } \because V_2 = \frac{q}{d_2}\right)$$

$$= \frac{q^2}{2g}\left[\frac{1}{d_1^2} - \frac{1}{d_2^2}\right] - (d_2 - d_1)$$

$$h_L = \frac{q^2}{2g}\left[\frac{d_2^2 - d_1^2}{d_1^2 d_2^2}\right] - \left[d_2 - d_1\right] \rightarrow (1)$$

But $q^2 = g\, d_1 d_2 \left(\frac{d_2 + d_1}{2}\right)$

Substituting the value of q^2 in equation ①, we get

$$\text{Loss of energy, } h_L = g d_1 d_2 \frac{(d_2+d_1)}{2} \times \frac{d_2^2-d_1^2}{2g d_1^2 d_2^2} - (d_2-d_1)$$

$$= \frac{(d_2+d_1)(d_2^2-d_1^2)}{4 d_1 d_2} - (d_2-d_1)$$

$$= \frac{(d_2+d_1)(d_2+d_1)(d_2-d_1)}{4 d_1 d_2} - (d_2-d_1)$$

$$= (d_2-d_1)\left[\frac{(d_2+d_1)^2}{4 d_1 d_2} - 1\right]$$

$$= (d_2-d_1)\left[\frac{d_2^2+d_1^2+2d_1d_2-4d_1d_2}{4 d_1 d_2}\right]$$

$$= (d_2-d_1)\left[\frac{(d_2-d_1)^2}{4 d_1 d_2}\right]$$

$$h_L = \frac{(d_2-d_1)^3}{4 d_1 d_2}$$

Problem

1. The depth of flow of water, at a certain section of a rectangular channel of 4m wide, is 0.3m. The discharge through the channel is $3 m^3/s$. Determine whether a hydraulic jump will occur, and if so. Find its height and loss of energy per kg of water.

Given data:

Depth of flow, $d_1 = 0.3m$

Width of channel, $b = 4m$

Discharge, $Q = 3 m^3/s$

Discharge per unit width, $q = \frac{Q}{b} = \frac{3}{4} = 0.75 m^2/s$

To find:

Height and loss of energy per kg of water

Formula Used.

$$d_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}, \quad \text{height} = d_2 - d_1, \quad h_L = \frac{(d_2 - d_1)^3}{4 d_1 d_2}$$

Solution:

Hydraulic jump will occur if the depth of flow on the upstream side is less than the critical depth on upstream side & if the froude number on the upstream side is more than one. critical depth is given by

$$d_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{0.75^2}{9.81}\right)^{\frac{1}{3}} = 0.385$$

Now the depth on the upstream side is 0.3m

This depth is less than critical depth and hence hydraulic jump will occur. The depth of flow after hydraulic jump is given by

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

$$= -\frac{0.3}{2} + \sqrt{\frac{0.3^2}{4} + \frac{2\times 0.75^2}{9.81\times 0.3}}$$

$$= 0.4862\ m$$

∴ Height of hydraulic jump $= d_2 - d_1 = 0.4862 - 0.3 = 0.1862 m$.

Loss of energy per kg of water due to hydraulic jump is given by equation as

$$h_L = \frac{(d_2-d_1)^3}{4d_1d_2} = \frac{(0.4862-0.3)^3}{4\times 0.4862\times 0.30}$$

$$= \frac{0.1862^3}{4\times 0.4862\times 0.3}$$

$$= 0.01106\ m\text{-}kg/kg$$

Result:

Height of hydraulic jump per kg of water $= 0.1862$ m.

Loss of energy per kg of water $= 0.01106$ m-kg/kg.

Specific Energy:

(1)

→ The Energy of a flowing liquid per unit weight is given by

$$\text{Total Energy, T.E} = z + h + \frac{v^2}{2g}.$$

where z = Height of the bottom of channel & above datum

h = ~~Depth~~ Depth of liquid

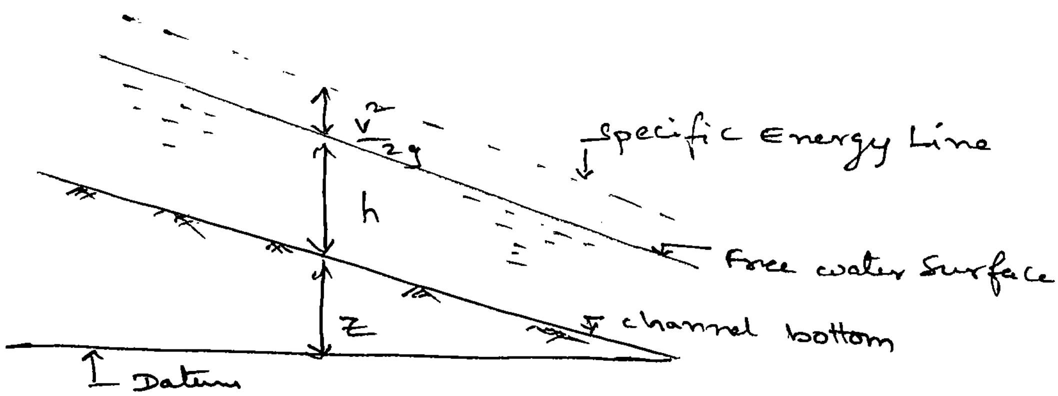
V = mean velocity of flow

→ If the channel bottom is taken as the datum as shown in figure, then the total energy per unit weight of liquid will be

$$E = h + \frac{v^2}{2g} \longrightarrow (1)$$

→ Derived equation shows the specific Energy. Hence

Specific Energy of a flowing liquid is defined as energy per unit weight of a liquid with respect to the bottom of the channel.



Specific Energy Curve: It is defined as the curve which shows the variation of specific energy with depth of flow.

It is obtained as:-

from equation ①, the specific energy of a flowing liquid

$$E = h + \frac{V^2}{2g} = E_p + E_k$$

Where E_p = Potential Energy of Flow = h

E_k = kinetic Energy of flow = $\frac{V^2}{2g}$

Consider a rectangular channel in which a steady but non-uniform flow is taking place.

Let Q = Discharge through the channel

b = Width of the channel

h = Depth of flow, and ($\because Q$ & b are constants)

q = Discharge per unit width

Then $q = \frac{Q}{\text{width}} = \frac{Q}{b}$ = constant

Velocity of flow, $V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{b \times h} = \frac{q}{h}$ $\left(\because \frac{Q}{b} = q\right)$

Substituting the values of V in equation ①, we get

$$E = h + \frac{V^2}{2g} = h + \frac{q^2}{2gh^2} = E_p + E_k \longrightarrow ②$$

Equation ②, gives the variation of specific energy (E) with the depth of flow (h). Hence for a given discharge Q, for different values of depth of flow, corresponding values of E may be obtained. Then a graph between specific energy (along x-x axis) and depth of flow, h (along y-y axis) may be plotted.

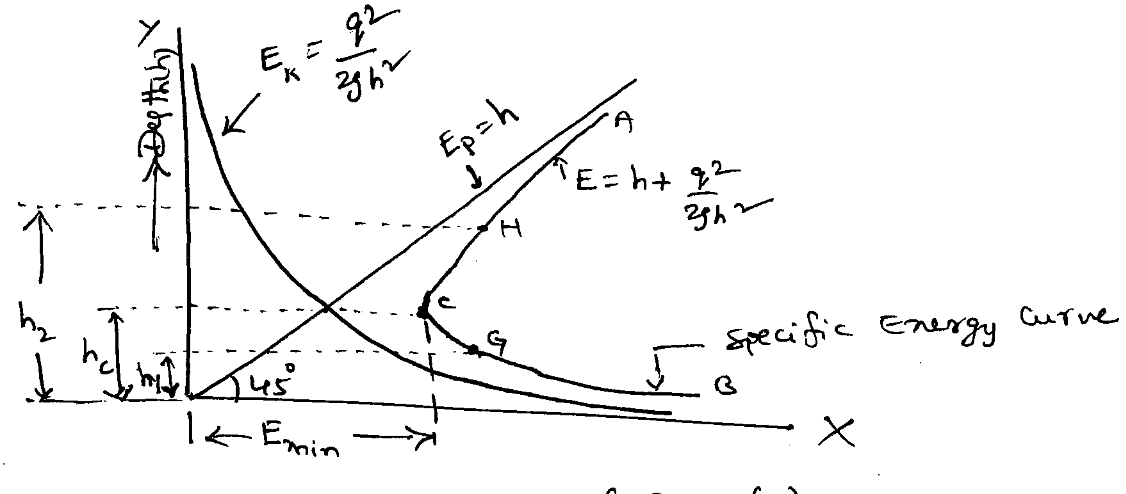


Fig: 1

The specific energy curve may also be obtained by first drawing a curve for potential energy (i.e $E_p = h$), which will be a straight line passing through the origin, making an angle of 45° with the x-axis as shown in fig 1.

Then drawing another curve for kinetic energy (i.e, $E_k = \frac{q^2}{2gh^2}$) which will be a parabola as shown in fig 1. By combining these two curves, we can obtain the specific energy curve. In figure ①, curve ACB denotes the specific energy curve.

Critical Depth (h_c): It is defined as the depth of flow of water at which the specific energy is minimum. This is denoted by 'h_c'. In Figure ①, Curve ACB is a Specific Energy Curve and point C corresponds to the minimum specific Energy. The depth of flow of water at C is known Critical depth.

The mathematical expression for critical depth is obtained by differentiating the specific energy Equation w.r.t. depth of flow & Equating the same to zero.

$$\frac{dE}{dh} = 0 \qquad \text{where} \quad E = h + \frac{q^2}{2gh^2} \text{ from Eq ②}$$

$$\frac{d}{dh}\left[h + \frac{q^2}{2gh^2}\right] = 0$$

$$1 + \frac{q^2}{2g}\left[-\frac{2}{h^3}\right] = 0$$

$$1 - \frac{q^2}{gh^3} = 0 \Rightarrow 1 = \frac{q^2}{gh^3} \Rightarrow h^3 = \frac{q^2}{g} \Rightarrow h = \left(\frac{q^2}{g}\right)^{1/3}$$

But when specific energy is minimum, depth is critical and it is denoted by h_c. Hence critical depth is

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \rightarrow ③$$

Critical Velocity (V_c): The velocity of flow at the critical depth is known as Critical Velocity V_c.

The mathematical expression for Critical Velocity is obtained from Equation ③ as

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

Taking cube to both sides, we get $h_c^3 = \frac{q^2}{g}$

$$g h_c^3 = q^2 \rightarrow (i)$$

But q = Discharge per unit width $= \frac{Q}{b}$

$$q = \frac{\text{Area} \times V}{b} = \frac{b \times h \times V}{b} = h \times V = h_c \times V_c$$

Substituting this value of 'q' in eq (i),

$$g h_c^3 = h_c^2 \times V_c^2$$

$$g h_c = V_c^2 \Rightarrow V_c = \sqrt{g \times h_c} \longrightarrow (4)$$

Minimum Specific Energy in terms of Critical Depth:

Specific Energy Eq is given by equation ②

$$E = h + \frac{q^2}{2gh^2}$$

When specific energy is minimum, depth of flow is critical and hence the above equation becomes as

$$E_{min} = h_c + \frac{q^2}{2g h_c^2} \longrightarrow (ii)$$

But from Equation (3), $h_c = \left(\frac{q^2}{g}\right)^{1/3}$

$$\therefore \ h_c^3 = \frac{q^2}{g}$$

Substituting the value of $\frac{q^2}{g} = h_c^3$ in equation (ii), we get

$$E_{min} = h_c + \frac{q^2}{2gh_c^2}$$

$$E_{min} = h_c + \frac{h_c^3}{2h_c^2} = h_c + \frac{h_c}{2} = \frac{3h_c}{2}$$

$$\boxed{\therefore E_{min} = \frac{3h_c}{2}} \longrightarrow (5)$$

Problem 1: Find the specific energy of flowing water through a rectangular channel of width 5m when the discharge is $10 \, m^3/s$ and depth of water is 3m.

~~Solution~~ Given Data:

Width of rectangular channel, $b = 5m$

Depth of water, $h = 3m$

Discharge, $Q = 10 \, m^3/s$

To find:

Specific Energy of flowing water

Formula Used :

Specific Energy Equation is, $E = h + \frac{V^2}{2g}$

Solution :

$$V = \frac{Q}{Area} = \frac{10}{b \times h} = \frac{10}{5 \times 3} = \frac{2}{3}$$

$$\therefore E = h + \frac{V^2}{2g}$$

$$= 3 + \frac{\left(\frac{2}{3}\right)^2}{2 \times 9.81}$$

$$= 3 + 0.0226$$

$$\therefore E = 3.0226\, m$$

Result :

Specific Energy of flowing water, $E = \underline{3.0226}\, m$.

Problem 2 : Find the critical depth and critical velocity of the water flowing through a rectangular channel of width 5m, when discharge is $15\, m^3/s$.

Given Data :
Width of channel, $b = 5m$
Discharge, $Q = 15\, m^3/s$

To find : Critical depth(h_c) & critical Velocity (V_c).

Formula Used:

$$\text{Critical Depth, } h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$\text{Critical Velocity, } V_c = \sqrt{g \times h_c}$$

Solution:

$\therefore$ Discharge per unit width, $q = \frac{Q}{b} = \frac{15}{5} = 3\ m^2/s$

$$\therefore \text{ critical Depth, } h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{3^2}{9.81}\right)^{\frac{1}{3}}$$

$$\therefore h_c = 0.972\,m$$

$$\therefore \text{ critical velocity, } V_c = \sqrt{g \times h_c}$$

$$= \sqrt{9.81 \times 0.972}$$

$$\therefore V_c = 3.088\ m/s$$

Result:

critical Depth of flow $= 0.972\,m$

critical Velocity of flow $= 3.088\ m/s$

Problem 3:- The discharge of water through a rectangular channel of width 8m, is $15\ m^3/s$ when depth of flow of water is 1.2 m. Calculate (i) Specific energy of flowing water, (ii) Critical Depth & Critical Velocity (iii) Value of Minimum Specific Energy.

Given Data:

Discharge, $Q = 15\ m^3/s$

Width, $b = 8m$

Depth, $h = 1.2m$

To find:

(i) Sp. Energy of flowing water

(ii) Critical Depth and Critical Velocity

(iii) Value of Minimum Sp. Energy.

Formula Used:

(i) $E = h + \frac{V^2}{2g}$

(ii) $h_c = \left(\frac{q^2}{g}\right)^{1/3}$ & $V_c = \sqrt{g \times h_c}$

(iii) $E_{min} = \frac{3h_c}{2}$

Solution: Discharge per unit width, $q = \frac{Q}{b} = \frac{15}{8} = 1.875\ m^2/s$

Velocity of flow, $V = \frac{Q}{Area} = \frac{15}{b \times h} = \frac{15}{8 \times 1.2} = 1.56\ m/s$

(i) Sp. Energy (E) is $E = h + \frac{V^2}{2g} = 1.2 + \frac{1.56^2}{2 \times 9.81} =$

(ii) Critical Depth, $h_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{1.875^2}{9.81}\right)^{1/3} = 0.71\ m$

Critical Velocity, $V_c = \sqrt{g \times h_c} = \sqrt{9.81 \times 0.71} = 2.639\ m/s$

(iii) Min. Sp. Energy, $E_{min} = \frac{3h_c}{2} = \frac{3 \times 0.71}{2} = 1.065\ m$

Critical Flow: It is defined as that flow at which the Specific Energy is minimum (or) The flow corresponding to critical depth is known as Critical flow.

From Equation (4) gives the relation for Critical Velocity in terms of Critical Depth as

$$V_c = \sqrt{g \times h_c}$$

$$\frac{V_c}{\sqrt{g \times h_c}} = 1 \qquad \text{Where } \frac{V_c}{\sqrt{g h_c}} = \text{Froude Number}$$

$\therefore$ Froude Number, $F_e = 1.0$ for critical Flow.

Sub-critical Flow & Streaming Flow or Tranquil Flow:-

When the depth of flow in a channel is greater than critical depth (h_c), the Flow is said to be Sub-Critical Flow. For this type of flow, the Froude Number is less than one i.e., $Fe < 1.0$

Super-Critical Flow &, Shooting Flow &, Torrential Flow:

When the depth of flow in a channel is Less than Critical depth, the flow is said to be super-critical flow, for this type of Flow $Fe > 1.0$

Alternate Depths: In the Specific Energy curve in Fig. 2, the point c corresponds to the minimum specific energy and the depth of flow at c is called Critical Depth. For any other value of Specific Energy, there are two depths, one greater than the critical depth and other smaller than the critical depth. These two depths for a given specific energy are called the alternate depths. These depths are shown as h_1 and h_2 in Fig. 2. (Or) the depth corresponding to points G and H in Fig. 2 are called alternate depths.

Condition for Maximum Discharge for a Given Value of Sp. Energy:

The Sp. Energy (E) at any Section of a channel is given by eq. ① as $E = h + \frac{V^2}{2g}$, where $V = \frac{Q}{A}$

$\therefore V = \frac{Q}{b \times h}$

$$\therefore E = h + \frac{Q^2}{b^2 \times h^2} \times \frac{1}{2g}$$

$$E = h + \frac{Q^2}{2g\, b^2 h^2}$$

$$Q^2 = (E-h)\, 2g\, b^2 h^2$$

$$\therefore Q = \sqrt{(E-h)\, 2g b^2 h^2} = b\sqrt{2g\left(Eh^2 - h^3\right)}$$

for max. discharge, Q, the expression $\left(Eh^2 - h^3\right)$ should be maximum.

$$\therefore \frac{d}{dh}\left(Eh^2 - h^3\right) = 0 \qquad \left[\because E \text{ is constant}\right]$$

$$2Eh - 3h^2 = 0$$

$$2E - 3h = 0$$

$$\therefore h = \frac{2}{3}E$$

$$E = \frac{3h}{2} \longrightarrow (iii)$$

But from eq. (5) Specific energy is minimum when it is equal to $\frac{3}{2}$ times the value of depth of critical flow. Here the eq (iii), the specific energy (E) is equal to $\frac{3}{2}$ times the depth of flow, thus equation (iii) represents the min. specific energy & h is the critical depth. Hence the condition for max. discharge for given value of sp. Energy is that the depth of flow should be critical.

Problems on UNIT-I (1)

Most Economical Trapezoidal channel

(1) An open channel of most economical section, having the form of a half hexagon with horizontal bottom is required to give a maximum discharge of 20.2 m^3/s of water. The slope of the channel bottom is 1 in 2500. Taking Chezy's constant, c=60 in Chezy's equation, determine the dimensions of the Cross-Section.

Sol:- Given Data:

Max. Discharge, $Q = 20.2\ m^3/s$

Bed slope, $i = \frac{1}{2500}$

chezy's Constant, $c = 60$

Channel is the form of a half hexagon as shown in fig. 1. This means that the angle made by the sloping side with horizontal will be 60°

$\therefore \tan\theta = \tan 60^\circ = \sqrt{3} = \frac{1}{n} \Rightarrow n = \frac{1}{\sqrt{3}}$

To be find:

Dimensions of the channels

Formula used:

$$\frac{b+2nd}{2} = d\sqrt{n^2+1}, \quad m = \frac{d}{2}$$

$$A = (b+nd)d, \quad Q = AC\sqrt{mi}$$

Solution: Let b = Width of the channel

d = depth of the flow.

Let, As the channel given is of most economical section, hence the condition given by Eq (1) and Eq. (2) should be satisfied i.e

Half of top width = One of the sloping side of

Hydraulic Meandepth = half of depth of flow.

from Eq. (1), $\frac{b+2nd}{2} = d\sqrt{n^2+1}$

For $n = \frac{1}{\sqrt{3}}$, $\frac{b+2\times\frac{1}{\sqrt{3}}\times d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1}$

$$\frac{\sqrt{3}.b+2d}{2\sqrt{3}} = \frac{2d}{\sqrt{3}}$$

$$\frac{\sqrt{3}.b + 2d}{2} = 2d$$

$$b = \frac{2\times 2d - 2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}} \longrightarrow (i)$$

Area of flow, $A = (b+nd)\,d$

$$A = \left(\frac{2d}{\sqrt{3}} + \frac{d}{\sqrt{3}}\right)d = \frac{3d^2}{\sqrt{3}} = \sqrt{3}\,d^2$$

From Eq. (2), $m = \frac{d}{2}$

For discharge, $Q = AC\sqrt{mi}$

$$20.2 = \sqrt{3}.d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{2500}}$$

$$d^{5/2} = \frac{20.2}{1.4696} = 13.745$$

$$\therefore d = (13.74)^{2/5} = \underline{\underline{2.852\,m}}$$

Substituting this value in Eq (i), we get

$$b = \frac{2d}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times 2.852$$

$$\therefore b = 3.293\,m$$

Result:

Width of the channel, $b = 3.293\,m$

Depth of flow, $d = 2.852\,m$

(2) A Power Canal of trapezoidal section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given, discharge equal to $14\,m^3/s$, bed slope 1:2500 and Manning's $N = 0.020$.

Given Data :-

Discharge , $Q = 16\ m^3/s$

Bed slope , $i = \frac{1}{2500}$

Manning's , $N = 0.020$

For excavation of the canal at the least cost, the trapezoidal section should be most economical. Here side slope (i.e value of n) is not given . Hence the best side slope for most economical trapezoidal section (i.e, the value of n) is given by equation as $n = \frac{1}{\sqrt{3}}$.

Let . b = width of channel , d = depth of flow

Solution: For most economical section,

Half of top width = Length of one of sloping side

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

For $n = \frac{1}{\sqrt{3}}$, $\frac{b + 2\cdot\frac{1}{\sqrt{3}}\cdot d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$ $\left[\because b = \frac{2d}{\sqrt{3}},\ n = \frac{1}{\sqrt{3}}\right]$

$$\frac{b + \frac{2}{\sqrt{3}}d}{2} = \frac{2d}{\sqrt{3}}$$

$$b + \frac{2d}{\sqrt{3}} = 2 \times \frac{2d}{\sqrt{3}}$$

$$b = \frac{2 \times 2d}{\sqrt{3}} - \frac{2d}{\sqrt{3}} = \frac{2d}{\sqrt{3}}$$

Area of trapezoidal section,

$$A = (b + nd)\,d = \left(\frac{2d}{\sqrt{3}} + \frac{1}{\sqrt{3}}d\right)d = \sqrt{3}\,d^2.$$

Hydraulic mean depth for most economical section, $m = \frac{d}{2}$

Now discharge, Q is given by $Q = AC\sqrt{mi}$

where,
$C = \frac{1}{N} \cdot m^{1/6}$

$$\therefore Q = \sqrt{3} \cdot d^2 \times \frac{1}{N} \cdot m^{\frac{1}{6}} \cdot \sqrt{m \times \frac{1}{2500}}$$

$$14 = \sqrt{3} \cdot d^2 \times \frac{1}{0.020} \times m^{\frac{1}{6} + \frac{1}{2}} \times \sqrt{\frac{1}{2500}}$$

$$14 = 1.732\, d^2 \times m^{2/3}$$

$$14 = 1.732\, d^2 \times \left(\frac{d}{2}\right)^{\frac{2}{3}}$$

$$14 = \frac{1.732}{2^{\frac{2}{3}}} \cdot d^{8/3}$$

$$14 = 1.09\, d^{8/3}$$

$$\therefore d^{\frac{8}{3}} = \frac{14.0}{1.09} = 12.844$$

$$d = (12.844)^{3/8} = (12.844)^{0.375}$$

$$\therefore d = 2.605\,m$$

$$b = \frac{2d}{\sqrt{3}} = \frac{2 \times 2.605}{\sqrt{3}} = 3.008\,m$$

<u>Result</u>: Width of channel, $b = 3.008\,m$

Depth of flow, $d = 2.605\,m$.

Problem on circular section

* The rate of flow of water through a circular channel of diameter 0.6 m is 150 litres/s. Find the slope of the bed of channel for maximum velocity. Take $C=60$.

Sol:

Given,

Discharge $Q = 150$ litres/s $= 0.15$ m^3/s

Dia. of channel, $D = 0.6$ m

value $c = 60$

let the slope of bed of channel $= i$

For max. velocity through a circular channel, depth of flow is given by,

$$d = 0.81 * D = 0.81 * 0.6 = 0.486 \text{ m}$$

$$\theta = 128°45'$$

$$\theta = 128.75 * \frac{\pi}{180} = 2.247 \text{ radians.}$$

hydraulic mean depth for max. velocity is,

$$m = 0.3 * D = 0.3 * 0.6 = 0.18 \text{ m}$$

wetted perimeter for circular pipe is,

$$P = 2R\theta = D * \theta$$

$$= 0.6 * 2.247 = 1.3482 \text{ m}$$

But $m = \frac{A}{P} = 0.18$ m

Area, $A = P * 0.18 = 0.18 * 1.3482$

$$= 0.2426 \text{ m}^2$$

For discharge, $Q = AC\sqrt{mi}$

$$0.15 = 0.2426 * 60 * \sqrt{0.81 * i}$$

$$= 6.175\sqrt{i}$$

$$i = \left(\frac{0.15}{6.175}\right)^2 = \frac{1}{1694.7}.$$

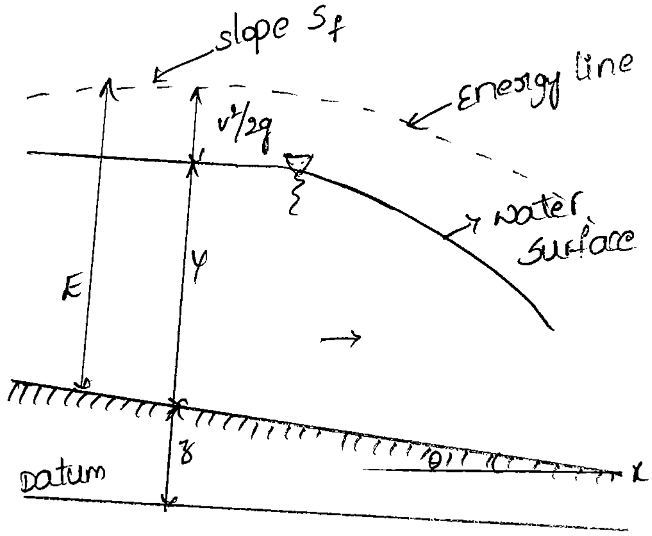
∴ Bed slope is 1 in 1694.7.

UNIT-II

* Explain direct step method.

A. Differential eq^n for GVF

* Consider the total Energy H of a gradually varied flow in a channel of small slope



$$H = Z + E$$
$$= Z + y + \frac{v^2}{2g}$$

where,

E = specific energy.

* Since water surface gradually varies in the longitudinal (x) direction, the depth of flow and total Energy are functions of x.

Differentiating above eq^n w.r.t x

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \left(\frac{v^2}{2g}\right)\frac{d}{dx}$$

1. $\frac{dH}{dx}$ represents the slope of Energy line (S_f) and slope of energy line decreases in the direction of motion. It is common to consider the slope of the decreasing line as positive.

$$\frac{dH}{dx} = -S_f$$

2. $\frac{dH}{dx}$ represents the S_0 slope of bed

$$\frac{dz}{dx} = -S_0$$

3. $\frac{dy}{dx}$ represents the water surface slope relative to the bottom of the channel.

4. $\frac{d}{dx}\left(\frac{v^2}{2g}\right)$ represents the slope of velocity head.

Differential Energy equation

We know that $H = E + Z$

Differentiating w.r.t x.

$$\frac{dH}{dx} = \frac{dE}{dx} + \frac{dZ}{dx}$$

$$-S_f = -S_0 + \frac{dE}{dx}$$

$$\frac{dE}{dx} = S_0 - S_f$$

Direct step method :-

* This method is the simplest and suitable for prismatic channels. From differential energy eq^n of GVF

$$\frac{dE}{dx} = S_0 - S_f$$

writing above eq^n in finite difference form

$$\frac{\Delta E}{\Delta x} = S_0 - \bar{S}_f$$

where,

$\bar{S}_f$ = average friction slope in reach Δx.

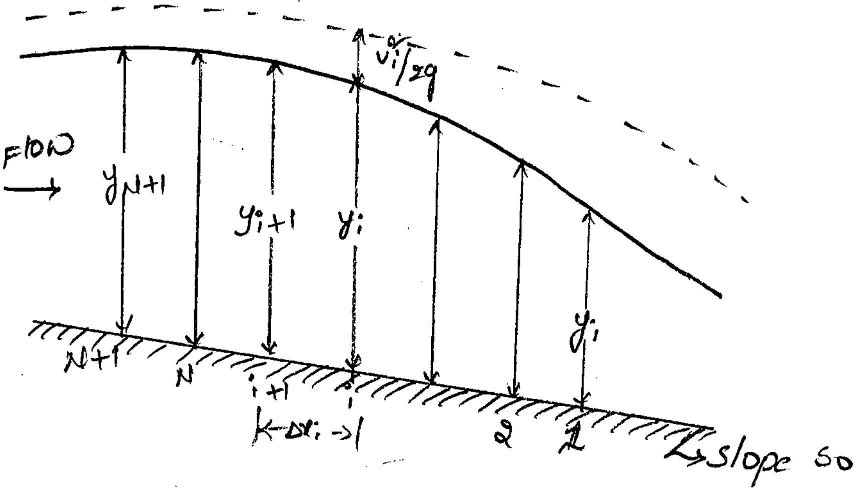
$$\Delta x = \frac{\Delta E}{S_0 - \bar{S}_f}$$

Between section ① & ②

$$x_2 - x_1 = \Delta x = \frac{E_2 - E_1}{S_0 - \frac{1}{2}(\bar{S}_{f_1} - \bar{S}_{f_2})}$$

procedure:

* Let it be required to find the water surface profile between 2 section 1 and (N+1), where the depths are y_1 & y_{N+1} respectively.
* channel reach is divided into N parts of known depths. i.e., values of y_i, $i=1, N$ are known.
* our requirements is to find distance Δx, b/w y_i and y_{i+1}.



* Now between 2 sections i and i+1

$$\Delta E = \Delta\left(y + \frac{v^2}{2g}\right)$$

$$= \Delta\left(y + \frac{Q^2}{2gA^2}\right)$$

$$\Delta E = E_{i+1} - E_i$$

$$\Delta E = \left[y_{i+1} - \frac{Q^2}{2g\,A_{i+1}^2}\right] - \left[y_i + \frac{Q^2}{2g\,A_i^2}\right]$$

$$\bar{S}_f = \frac{1}{2}\left[\bar{S}_{f_{i+1}} + \bar{S}_{f_i}\right] \qquad \left(\because S_f = \frac{n^2Q^2}{A^2R^{4/3}}\right)$$

$$\bar{S}_f = \frac{n^2Q^2}{2}\left[\frac{1}{A_{i+1}^2\,R_{i+1}^{4/3}} + \frac{1}{A_i^2\cdot R_i^{4/3}}\right]$$

Sub values $(E_{i+1} - E_i)$ and $\bar{S}_f$ in Eq^n $\Delta x = \dfrac{E_{i+1} - E_i}{S_0 - \bar{S}_f}$, we can calculate the value of Δx_i

* The sequential Evalution of Δx_i starting from $i=1$ to N, will given the distance between the N sections & hence the GVF prorofile.

* In a gradually varied flow a rectangular channel of bottom width 3.0 m the discharge is 8 m^3/s and the depth of flow changes from 1.4m at section M to 1.05m at section N. calculate the average energy slope between these 2 section. Assume $n=0.018$.

Sol: Given data.

width of channel $b = 3\,m$

At s/c M. depth of channel $d_1 = 1.4m$

discharge $Q = 8\,m^3/s$

At section N depth of channel $d_2 = 1.05\,m$

$n = 0.018$

In gradually varied flow the manning's formula is given for any section as $V = \frac{1}{n} R^{2/3} S_f^{1/2}$

where, S_f = Energy slope at that section

Hence $S_f = \frac{n^2 V^2}{R^{4/3}}$

Average Energy slope between 2 section

$$\bar{S_f} = \frac{S_{f_1} + S_{f_2}}{2}$$

property	Section M	Section N
A	$b*d_1 = 3*1.4 = 4.2\,m^2$	$3*1.05 = 3.15\,m^2$
P	$b+2d = 3+2(1.4) = 5.8\,m$	$3+2(1.05) = 5.10\,m$
R	$A/P = 4.2/5.8 = 0.724\,m$	$3.15/5.10 = 0.6176\,m$
V	$Q/A = 8/4.2 = 1.9048\,m/s$	$8/3.15 = 2.5397\,m/s$
S_f	$\frac{n^2 V^2}{R^{4/3}} = \frac{(0.018)^2 * (1.9048)^2}{(0.724)^{4/3}} = 1.8082 \times 10^{-3}$	$= \frac{0.018^2 * 2.5397^2}{(0.6176)^{4/3}} = 3.973 \times 10^{-3}$

$$\bar{S_f} = \frac{1.8082 \times 10^{-3} + 3.973 \times 10^{-3}}{2} = 2.890 \times 10^{-3}$$